Is All Active Learning the Same? Exploring Mathematically Derived Inquiry-Based Learning

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This article explores inquiry-based learning (IBL) pedagogy, particularly in mathematics education, examining how it differs from problem-based learning (PBL) and case-based learning (CBL). IBL is defined as a student-centered approach involving sequenced problems or tasks that build engagement and understanding through group work. While IBL, PBL, and CBL share characteristics like problem solving and self-directed learning, each has distinct practices. The article presents a mathematics education community definition of IBL and provides detailed comparisons and examples to help readers understand IBL's potential for broad application in scholarship beyond mathematics education.

Introduction

Educators new to the profession often model their teaching on the ways they themselves were taught rather than on the approaches emphasized in their preparation or professional development. This is especially true when their own learning experiences relied heavily on traditional, lecture-based instruction, even if they have since been introduced to more active or student-centered methods. Our intent with this article is to clarify the principles and practices of inquiry-based learning (IBL), distinguish it from related approaches such as problem-based and case-based learning, and illustrate how these distinctions can inform effective teaching across educational contexts.

The pedagogy we explore in this article is inquiry-based learning (IBL). If one were to break down the word into its parts, *inquiry* is the descriptor of the instructional strategy. However, the use of the term can vary significantly in different fields of study. For example, the medical education field has used IBL as an encompassing term for types of active learning (Haq, 2017). In the field of physics education, inquiry-based learning is often applied to enhance students' understanding of fundamental concepts. It involves hands-on experimentation, data collection, and critical thinking to foster deeper comprehension (Smith et al., 2020). Some researchers in the field of mathematics education use IBL in a slightly different way, which we will describe and expand on. This alternative method can be used beyond mathematics education in all curriculum and instruction and educational research areas. In this article, we share an additional way of characterizing inquiry-based learning that can be used by both educators and researchers alike.

As mentioned, IBL is often used as an umbrella term to describe active learning pedagogy in teaching and learning. In some fields, IBL has been used interchangeably with multiple forms of active learning, such as problem-based learning (PBL) and case-based learning (CBL), among others (Verduin et al., 2013). However, we have identified a scenario in which IBL is not necessarily interchangeable with PBL or CBL. We first discuss active learning in general and then the similarities and differences between the PBL, CBL, and IBL pedagogies, focusing on IBL. We also depict how IBL can look in the mathematics classroom (how we use it) to inform the literature on teaching with this method. Although this article takes more of a historical perspective in its framing, we also discuss recent advancements in PBL, CBL, and IBL definitions and implementation in the classroom.

Method

To develop a comprehensive understanding of the pedagogies under investigation—problem-based learning (PBL), case-based learning (CBL), and inquiry-based learning (IBL)—we conducted a systematic review of the relevant literature. Each of us authors independently examined peer-reviewed articles, books, and conference papers that explored the theoretical foundations, practical implementations, and documented outcomes of these three approaches. As we reviewed the literature, we critically analyzed key themes and patterns, focusing on how PBL, CBL, and IBL were defined, applied, and assessed across diverse educational contexts. We each summarized our findings to highlight how these pedagogies have evolved and how they continue to shape instructional practice.

After completing our individual reviews, we came together to discuss and synthesize our findings. In these collaborative discussions, we shared insights from our independent analyses and compared the defining features, instructional practices, and outcomes of PBL, CBL, and IBL. Through this comparative analysis, we identified both common threads and important distinctions among the three approaches. These conversations allowed us to integrate our perspectives and develop a more cohesive and nuanced understanding of how the pedagogies intersect and diverge.

Drawing from this synthesis, we created Table 1 to illustrate the key components of PBL, CBL, and IBL. We began by identifying major categories for comparison, such as instructional strategies, student engagement, assessment methods, and educational outcomes. Then we populated the table with concise descriptions and representative examples of each instructional strategy. We used these examples to write accompanying descriptive paragraphs that show how the approaches are enacted in practice. Finally, we reviewed and refined the table through several iterative rounds of feedback to ensure clarity, accuracy, and comprehensiveness.

Although our analysis primarily takes a historical perspective to trace the evolution of these pedagogies, we also consider recent developments in their definitions and classroom implementation. In doing so, we situate PBL, CBL, and IBL within the broader tradition of active learning—an approach that stands in contrast to traditional didactic instruction. Active learning engages students directly in constructing their understanding rather than passively receiving information, and the pedagogies examined here represent distinct yet related ways of fostering that learning.

The method employed in this study aligns with the literature search approach commonly used in educational research. This approach involved reading, analyzing, and sorting literature to identify essential attributes and draw meaningful conclusions. It is characterized by non-structured qualitative analysis, which allows researchers to explore logical relationships and trends within the literature.

Active Learning

Active learning is an alternative to traditional didactic instruction (lecturing, passive learning). Definitions of active learning can differ based on the content (subject matter) being taught. However, it typically involves having learners engage more directly in their learning than the traditional "sit back and listen to your instructor feed you knowledge" type of instruction. Active learning has been broadly defined as "anything course-related that students in a class session are called on to do other than simply watching and listening to a lecture and taking notes" (Felder & Brent, 2016, p. 112). It is also defined as "any instructional method that engages students in the learning process. In short, active learning requires students to do meaningful learning activities and think about what they are doing" (Prince, 2004, p. 223). Both definitions of active learning are used widely in the literature. By either definition, however, the goal is learner-centered, meaningful learning.

Active learning relates to a constructivist viewpoint of situating learning in authentic contexts, learner responsibility for intentional learning, critical thinking/higher-order thinking, problem-solving skills, and providing learners with the context for *why they need to know this* (Doolittle et al., 2023; Grabinger & Dunlap, 1995). On a side note, mathematics is a discipline in which knowledge has often been contextualized as just working with numbers versus gaining practical knowledge and understanding *why* learners need to know how to think mathematically. This may be why the meaning of IBL has been such a focus in mathematics education for years. For more information, see Chu et al., 2021; Friesen & Scott, 2013; Laursen & Rasmussen, 2019; Levy et al., 2013.

There is general agreement that PBL, CBL, and IBL all fall under the umbrella of active learning (Hopper, 2018; Joseph et al., 2022). These teaching approaches share key characteristics, such as being student-centered, emphasizing collaborative group work, and fostering problem-solving skills for tackling complex, real-world issues. However, it is essential to note that IBL, CBL, and PBL each have distinct practices.

For instance, in an IBL session, the instructor might start by posing a challenging question or presenting a series of carefully sequenced problems. These are designed to help learners build new concepts from what they already know, often with little direct instruction.

Table 1
A Comparison of Problem-Based Learning, Case-Based Learning, and Inquiry-Based Learning

Characteristic	*PBL*	*CBL*	*IBL*
Role of Instructor	Limited guidance*	Active guidance*	To sequence problems (or a set of tasks) and facilitate learning
Role of Student	Take the initiative on defining the problem, research, and solution(s)	Discussion of the provided case and possible solution(s)	To be deeply engaged in learning the content and solving the problems/tasks
Problem Type	Open-ended problems (could be a case)	Case-based problems	Sequence of problems/tasks that lead to a big idea
Inquiry Style	Open inquiry*	Guided or structured inquiry*	Guided with questions/tasks provided by instructor
Learning Method	Self-directed*	Shared facilitator and self-directed*	Shared facilitator and self-directed
Equity**	Considered but students find their own resources to be successful	Considered but students find their own resources to be successful	An explicit element of the teaching & learning
Student Thinking	May or may not emerge via open discussions	Should emerge via discussions	Becomes explicit in the classroom though the learning process
Group Work	Small groups (4-8 students)*, all with different problems and solutions	Small groups* (3-5 students), all with same problem, then bring solutions and discussion to full group	Students working collaboratively in small groups (2-4 students), who then bring solutions and discussion to full group
Product	Student presentation(s)*	Wrap-up by instructor*	The sequence of problems/tasks lead to the understanding of concept, idea, or theorem.
Recent Advancements	Incorporating global challenges such as financial literacy and data-driven decision making.	Using digital simulations and case libraries to make CBL more engaging for students.	Collaboration among students and the role of instructors as facilitators.

Notes.
*Some PBL and CBL characteristics in Table 1 are adapted from Hopper (2018, p. 145). See her article for other PBL and CBL characteristics.
**General equity research was found, but not specifically within PBL and CBL literature.

Students then work together to understand and solve the problem, looking to the instructor for clarification. However, rather than providing direct answers, the instructor typically responds with another thought-provoking question, much like the Socratic method. In PBL, the instructor presents an open-ended, complex problem for students to solve. While students can ask questions, they are typically expected to research and explore potential solutions independently, with the instructor providing guidance as needed. CBL is similar to PBL but is more structured. In CBL, the problem is based on a specific case with constraints within which the students must work, offering a bit more guidance than in PBL.

Much of the literature describes using ill-structured problems, which are more challenging because they offer multiple solution paths, have vague or unclear goals and constraints, and involve different criteria for evaluating solutions (Tawfik & Jonassen, 2013, p. 386). An authentic problem might be based on a real-world event (Ryan, 2021). Whether in PBL, CBL, or IBL, groups work together throughout the duration of the problem or case, with each member contributing to the team's collective knowledge. This collaborative approach facilitates learning and makes knowledge acquisition a shared, rather than an individual, experience (Hmelo-Silver & DeSimone, 2013; Tyyskä et al., 2020). In this way, all three approaches use ill-structured problems, questions, and group work, keeping learners at the center of the process. Next, we dig deeper into these three pedagogies: PBL, CBL, and IBL.

Problem-based Learning

The origin of the term *problem-based learning* is commonly acknowledged to have emerged from Barrows and Tamblyn in the 1960s in a series of papers on medical education (Schmidt, 2012). The goals of PBL include constructing a knowledge base or case library, developing problem-solving skills through practice, fostering self-directed learning, and enhancing collaboration and motivation (Hmelo-Silver et al., 2007; Hmelo-Silver & DeSimone, 2013). Additional characteristics of PBL include the use of problematic examples or cases to be discussed and analyzed by teams using problem-solving and critical-thinking skills, and the more significant responsibility of learners compared to other teaching methods (Müller & Henning, 2017; National Library of Medicine Medical Subheading, 2023). In PBL, the roles of the learners and teacher are transposed. Learners can look to the teacher as a facilitator or guide, but they are the primary directors of their own knowledge gathering. Learners can construct mental models or schemas to synthesize existing knowledge to the newly learned material. Learners can develop the questions to pursue because PBL is less *guided* than other pedagogies. Although many teachers use elements of PBL in their classrooms, they often do so without a deep understanding of its theoretical underpinnings, resulting in variations in how the approach is implemented (Tawfik & Kolodner, 2016).

Recent Advancements in PBL in Mathematics

In recent years problem-based learning in mathematics has been increasingly recognized for its ability to foster critical thinking, creativity (Aba-Oli et al., 2024; Aini et al., 2019), and interdisciplinary application (Rézio et al., 2022). Recent research has emphasized the importance of integrating real-world problems to make mathematics more relevant and engaging for students (Arthur et al., 2018; Schoenherr, 2024). For example, PBL projects that incorporate global challenges like climate change (Cujba & Pifarré, 2024), financial literacy, or data-driven analysis have been shown to promote mathematical skills and problem-solving capabilities (Rodrigues, 2023). Challenges remain in implementing PBL effectively, because it requires a shift from traditional teaching methods to a more student-centered approach (Aksela & Haatainen, 2019; Yang et al., 2021). Studies from the past five years have also noted the importance of scaffolding, with instructors providing appropriate support to ensure students develop the necessary foundational skills to engage in self-directed problem solving (Abdullah & Girei, 2022; Siebörger, 2021). PBL has shown promise in preparing students for lifelong learning by encouraging them to take ownership of their education and confidently approach complex problems.

Examples of PBL in Use

Mathematics learning might not seem like an obvious fit for PBL because mathematical problems are typically well-structured and have specific correct answers—both characteristics that contrast with the open-ended, ill-structured nature of traditional PBL (Müller & Henning, 2017). In one study, Tawfik and Lilly (2015) explored PBL in a university mathematics course that integrated real-world, interdisciplinary problems within a flipped classroom framework. Students were encouraged to propose authentic contexts of interest—including social and psychological themes such as family counseling and domestic dynamics—and to apply mathematical reasoning and modeling to analyze these scenarios. Problems that they developed for the learners to solve were based on those topics. The researchers hypothesized that because the learners were interested in those topics, any prior knowledge about the issues would be activated during PBL activities. The study focused on learners' perspectives of PBL and did not cover instructors' perspectives. The researchers found that learners reported increased engagement and a stronger sense of relevance when mathematical concepts were applied to personally meaningful contexts. However, some students expressed uncertainty about how these interdisciplinary problems related to traditional mathematical learning objectives, suggesting that careful scaffolding is necessary when adapting PBL to mathematics.

Judging from the research, health sciences education appears to adopt PBL approaches frequently (Jin & Bridges, 2016; Trullàs et al., 2022). One such study explored the effectiveness of PBL compared to conventional learning in medical education (Preeti et al., 2013). The researchers found that students' pre/posttest scores improved significantly for those experiencing PBL methods compared to those experiencing traditional lectures. The PBL students appeared to improve their critical-thinking skills and communication between classmates, and they expressed overall positivity toward the PBL strategy (Preeti et al., 2013).

PBL has been used as a teaching method in a secondary musical instrument methods course for musicians training to become teachers (Blackwell & Roseth, 2018). The PBL activities included video assessments, written scenarios, performance scenarios, group activities, and structured peer teaching. Learners in that course reported that participating in PBL increased their confidence in solving problems independently and fostered greater flexibility and adaptability in their approach to problem solving.

Case-based Learning

Case-based learning is an educational strategy that has been debated and defined by researchers (Ameta et al., 2020; Naeimi et al., 2017; Thistlethwaite et al., 2012). Since its inception, a single definition of CBL has yet to be agreed on in the literature. CBL has been implemented in numerous academic subjects, including business, science, medicine, and law (Burgess et al., 2021; Kantar & Massouh, 2015; Yadav et al., 2014). Using cases that illustrate problems for learners to solve has been a teaching method in academic disciplines for over a century (Davis & Yadav, 2014). According to Thistlethwaite et al. (2012), James Lorrain Smith, an educator in pathology at the University of Edinburgh, is considered the originator of CBL, which is likely the oldest among the different types of active learning. In 1912, Smith referred to CBL as a case method for teaching (Thistlethwaite et al., 2012). Barrows and Tamblyn are considered the originators of the term *problem-based learning* in medical education in the 1960s. The term *case-based learning* emerged during the late 1970s to early 1980s in a discussion by Barrows and Tamblyn (1977) on how PBL was used with medical *patient* cases (Barrows, 1986). Researchers tend to accept CBL as a learner-centered approach in which the emphasis on knowledge acquisition is based on solving authentic, open-ended, ill-structured cases (Savery & Duffy, 2001).

Strategies common to case-based teaching methods include grouping learners in teams and presenting them with ill-structured, authentic, and complex problems to solve; facilitation is scaffolded; and preparatory work in varied amounts is assigned to encourage in-session interaction. Because CBL does not include the lecture method, required preparatory work is necessary for teams to participate in this active-learning style of teaching and learning. A difference between CBL and typical didactic learning can be described as follows: "If all of the information were given prior or during the session, without the need for inquiry, then the session would just be a lecture or reading" (McLean, 2016, p. 42).

Recent Advancements in CBL in Mathematics

Case-based learning in mathematics has evolved with the use of multimedia and interactive tools to present structured, real-life cases that encourage critical thinking and decision making (Angraini & Nurmaliza, 2022; Firdaus & Darari, 2024). Research from recent years has focused on using digital simulations and case libraries to make CBL more dynamic and engaging for students (Firdaus & Darari, 2024). In mathematics, this approach has been applied to scenarios such as statistical modeling, optimization problems, and systems analysis (Mustyala & Bisi, 2025), where students analyze cases and propose solutions within given constraints. A key insight from recent research is that CBL provides a structured learning environment (Burgess et al., 2021), which can be particularly beneficial for students who might struggle with the open-ended nature of PBL (Dewi & Nurjanah, 2022). The structured cases allow students to develop analytical reasoning while still benefiting from collaborative problem solving. Researchers have also emphasized the need to balance structure with opportunities for exploration to maximize learning outcomes. This balance ensures that students engage deeply with mathematical concepts while honing their ability to apply them to real-world situations (Dewi & Nurjanah, 2022).

An Example of CBL in Use

A field-based, quasi-experimental study by Han et al. (2013) investigated the effects of multimedia case-based learning (CBL) on preservice teachers' knowledge acquisition and integration related to technology use in education. The participants were 78 students enrolled in a teacher preparation course at a private university in South Korea. They were divided into two groups: One group engaged with video cases, while the other did not. For six weeks prior to the study, all participants learned about lesson planning, instructional models, and educational technology. During the latter part of the course, the researchers implemented the CBL intervention, integrating this foundational content with opportunities for practical application.

Participants n the CBL condition viewed carefully selected video clips that illustrated authentic classroom scenarios, including examples of lesson implementation, classroom layouts, and technology integration. They viewed one video per week over two weeks at the end of the course, followed by group discussions and a collaborative reflection paper. Participants in the comparison group received similar content through PowerPoint slide decks and written materials, such as sample syllabi, and completed the same discussion and reflection tasks without the videos.

Han et al. (2013) evaluated participants' group reflections and individual assessments to measure both knowledge acquisition and knowledge integration. Results indicated that the video-based CBL group demonstrated stronger conceptual understanding and greater ability to connect theoretical knowledge to practical classroom applications than those in the non-video condition.

All participants completed a pretest using selected items from the Technology, Pedagogy, and Content Knowledge (TPACK) survey to assess their existing understanding of and confidence with integrating technology into classroom practice. Lecture served as the primary instructional method for all participants until the seventh week of the course, just before the start of the two-week CBL intervention. Following the CBL activities—whether through video cases or PowerPoint slide decks—all participants completed a posttest version of the survey.

The posttest data were analyzed using analysis of covariance (ANCOVA), with intervention type as the independent variable and five categories from the TPACK framework (Koehler & Mishra, 2009; Mishra & Koehler, 2006) as dependent variables: technology knowledge (TK), pedagogical knowledge (PK), technological content knowledge (TCK), technological pedagogical knowledge (TPK), and the overall TPACK composite. Participants who engaged with the video-based cases demonstrated higher overall perceptions of their technological and pedagogical knowledge than those who reviewed only written materials. Significant differences were found between the two groups in the TK and PK categories. The researchers concluded that the multimedia CBL approach was more effective than lecture-based instruction for helping preservice teachers develop the skills and confidence needed to integrate technology into their teaching.

Inquiry-based Learning

Inquiry-based learning is an active learning approach in which learners, often working collaboratively, engage with a question or a carefully sequenced set of problems and use self-directed investigation to explore possible explanations or solutions (Lazonder, 2014; Lazonder & Harmsen, 2016). Through this process, learners generate questions, gather and analyze evidence, and construct meaning from their findings, becoming active agents in their own learning (Lazonder & Harmsen, 2016). The roots of IBL in mathematics education can be traced to the mid-20th century "Moore Method," developed by mathematician R. L. Moore, which emphasized student discovery and proof-writing through minimal direct instruction. Over time, this approach evolved into a more inclusive, collaborative, and student-centered model of inquiry. Modern IBL practices, particularly in mathematics, have been shaped by the work of Laursen and colleagues (Laursen et al., 2014; Laursen & Rasmussen, 2019), who have broadened the conception of IBL to focus on equitable participation, community building, and shared authority in the learning process. Laursen et al. articulate IBL through four interrelated pillars: (1) students engage deeply with meaningful mathematics, (2) students take responsibility for their own learning, (3) instructors use student thinking to guide lessons, and (4) students communicate their reasoning and collaborate with others. These pillars provide a framework for designing IBL experiences that balance structure and autonomy while emphasizing sense-making and discourse.

While IBL shares characteristics with PBL and CBL, such as emphasizing student-centered inquiry, it differs in both structure and instructional intent. In PBL, learning typically begins with a complex, authentic problem designed to guide learners toward specific, real-world outcomes. In CBL, instruction is organized around detailed, context-rich cases that illustrate professional practices or disciplinary principles. In contrast, IBL focuses on working through a deliberately sequenced set of problems to deepen conceptual understanding. These problems are not necessarily drawn from real-world contexts; rather, they are designed to help learners uncover and make sense of underlying ideas or relationships within the discipline.

Table 1, modified from Hopper (2018) to include IBL, demonstrates how our use of IBL is different from examples of PBL and CBL. We do not claim any active learning pedagogy to be better than another. Instead, we provide Table 1 as a means to frame teaching, learning, and research within active learning.

Recent Advancements of IBL in Mathematics

IBL in mathematics has gained momentum over the last few years, with research emphasizing its impact on deepening conceptual understanding and fostering collaboration among students (Beswick, 2021; Fry et al., 2025; Özturk et al., 2022). Studies highlight that IBL encourages students to engage actively in the learning process by exploring mathematical problems, formulating hypotheses, and collaboratively developing solutions (Laursen & Rasmussen, 2019). Recent advancements in IBL research have identified the role of instructors as facilitators who guide students through a sequence of well-designed tasks, enabling learners to construct knowledge gradually (Santos-Trigo, 2024). As mentioned previously, scholars such as Laursen and Rasmussen (2019) have expanded the definition of IBL to include an explicit focus on equity, suggesting that inquiry can help create inclusive classroom environments where diverse student voices are valued. This approach ensures that students not only learn mathematical concepts but also gain confidence in articulating and defending their reasoning in collaborative settings (Hidayat et al., 2024).

An Example of IBL in Use

A study by Anstey (2017) investigated authentic inquiry learning in an anatomy course to determine learners' experiences of learning through an inquiry project. In this context, authentic learning refers to opportunities that connect anatomical concepts to relevant professional practices. Like other researchers, Anstey noted that no single model of IBL applies universally and that the approach exists along a continuum from learner-driven to instructor-led. Within this framework, IBL requires balancing the degree of guidance provided by the instructor with the autonomy given to learners (Anstey, 2017; Lippmann, 2021).

Anstey's study specifically used qualitative methods to investigate learners' experiences with IBL in human gross anatomy projects. The main task for participant groups was to formulate an inquiry question of their choosing. Although the project was relatively open-ended, all three groups approached knowledge construction in similar ways. The study focused extensively on the learner experience, while giving little attention to the perspectives of instructors or facilitators.

Illustration of IBL

To provide a tangible representation of inquiry-based learning, we offer a mathematics example drawn from the professional experience of the first author. The objective of this lesson is for students to understand different ways geometric triangles can be classified. Although this example focuses on mathematics, a similar approach could be used in other disciplines, such as classifying species in biology or materials in environmental science. The sequence shown in Table 2 demonstrates how IBL emphasizes exploration, collaboration, and guided facilitation. Our IBL approach creates a learning environment where mathematical understanding emerges from the collective wisdom of the learners rather than being driven by individual solutions or predetermined outcomes, as often seen in PBL and CBL. This collaborative, inquiry-based approach deepens mathematical comprehension and develops students' abilities to communicate their thinking and appreciate diverse problem-solving perspectives.

Summary

Problem-based, case-based, and inquiry-based learning are active, student-centered pedagogies that have become popular across educational disciplines, especially in mathematics. These approaches share commonalities in promoting students' critical thinking, collaboration, and self-directed learning while engaging them with real-world, ill-structured problems. This article has outlined how IBL, as defined in mathematics education, distinguishes itself through its intentional sequencing of tasks to build conceptual understanding, its collaborative and inquiry-driven nature, its explicit focus on equity and inclusion, and its attention to the role of the instructor in the sequencing of tasks to help the learners build knowledge.

The common thread across all three pedagogies is active, student-centered learning methods that promote self-directed and collaborative problem-solving skills. All three are benefiting from recent advances in ease of access to real-life cases and data and technologies that make collaboration more effortless (even

Table 2
Illustration of Inquiry-Based Learning (IBL) in a Mathematics Context

Phase	*Instructor Actions*	*Student Actions*	*IBL Focus*
Exploration	Provide students with a variety of triangles differing in color, size, and orientation. Encourage open-ended sorting without specific instructions.	In small groups, students sort triangles according to self-chosen criteria and record their classification systems.	Promotes curiosity, observation, and inductive reasoning.
Discussion	Circulate among groups, asking probing questions such as "How did you decide to group these?" and "What similarities or differences do you notice?"	Students articulate and defend their reasoning, refine classifications, and consider alternative perspectives.	Encourages critical thinking, discourse, and collaboration.
Synthesis	Facilitate a whole-class discussion using examples from each group. Guide learners to recognize multiple valid classification systems (by sides, angles, or orientation).	Compare and refine classifications, discussing the usefulness and implications of different systems.	Deepens conceptual understanding and fosters shared meaning-making.

in online learning environments). Specifically, these approaches—PBL, CBL, and IBL—share several key characteristics:

- Active participation: All three methods emphasize active learning, where students engage with the material directly rather than passively receiving information through lectures.
- Problem solving: All three methods utilize problems or cases as a starting point for learning, fostering critical thinking and problem-solving skills.
- Self-directed learning: All three methods encourage learners to take charge of their own learning, seeking out and synthesizing information in a self-directed manner.
- Collaboration: Collaboration is critical, with students working in groups to discuss and solve the presented problems or cases.
- Real-world relevance: The problems or cases used in these methods are typically authentic, ill-structured, and open-ended, mirroring real-world complexity.
- Facilitator role: Instructors in these paradigms often act as facilitators or guides rather than traditional lecturers, providing support and scaffolding as needed.
- Adaptability and flexibility: These methods require students to adapt to new information and approaches, often resulting in increased confidence in their problem-solving abilities.
- Integration of knowledge: Learners are expected to integrate new knowledge with their existing knowledge base, fostering a deeper understanding of the material.
- Engagement and motivation: These methods aim to increase student engagement and motivation to learn by involving students in the learning process and focusing on topics of interest.
- Lifelong learning skills: These methods aim to impart specific knowledge and develop lifelong learning skills that will be valuable in students' future endeavors.

In sum, these methods differ from traditional didactic approaches, which are more instructor-led and often involve the direct transmission of information to students. Instead, PBL, CBL, and IBL place students at the center of the learning process, emphasizing discovery, analysis, and application over rote memorization.

One of the key limitations of this study is the specificity of our definition and application of IBL to mathematics education. While we have aimed to provide a broad understanding of IBL that can be adapted across different educational disciplines, the frameworks we have presented are heavily rooted in the context of mathematics. This focus may limit the direct applicability of our definitions to other disciplines, where the nature of inquiry and the structure of problems can differ significantly. Additionally, our discussion assumes a certain level of familiarity with active-learning principles, which might not be universally shared among all educators, particularly those new to these methods.

The implications of our definitions of IBL suggest that while it can be a powerful tool across educational settings, its effectiveness depends on the context in which it is implemented. Educators must carefully consider how the principles of IBL align with their subject matter and the specific learning goals of their students. Our nuanced definition may require adaptation to fit different disciplines, requiring educators to be flexible and creative in applying IBL strategies. The emphasis on equity and collaborative learning within our IBL framework highlights the need for intentional instructional design to ensure that all students can engage meaningfully with the material. We encourage educators and researchers to think about how they can integrate these aspects of IBL into their teaching practices and research.

In summary, inquiry-based learning, particularly as rooted in mathematics education, distinguishes itself from problem-based learning and case-based learning through its intentional sequencing of tasks that guide learners toward conceptual understanding. While all three approaches fall under the active learning umbrella, they differ in how they frame problems and in the respective roles of the instructor and the learner. IBL emphasizes the collective construction of knowledge through guided inquiry, where both students and instructors engage collaboratively in sense-making. This evolving form of IBL provides a model of learning that values curiosity, communication, and community. Although its foundations lie in mathematics, the principles of IBL have broad potential for application across disciplines. We hope that the distinctions and definitions presented here support educators and researchers in examining and enriching the teaching and learning processes within their own contexts.

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Declaration of Interest Statement

The authors declare they have no known relevant financial or non-financial competing interests to report.

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