

The Definition of the Derivative

Purpose. To reinforce the procedure for calculating the derivative of a function at a point, using the definition of the derivative, and to encourage students to critically evaluate the algebraic simplifications involved in such a calculation.

Preparation (before class) and implementation (in class). This activity is designed to be done by each student individually, before they share their solution(s) with the other students in their group of 2 or 3 students. Specifically, each student should work on Exercise 1 for 4 or 5 minutes, and then discuss it with the other students in their group (after which the instructor should debrief the entire class on the correct order for the algebraic steps in the calculation). This same process should be successively followed for each of the other exercises—Exercises 2, 3, and 4.

If you wish to make this project as discovery-based as possible, you can distribute the activity, or have it waiting on students' tables as they come in, without instructions. Alternatively, you can provide directions like the following on each of the exercises:

- Exercise 1: “The steps for deriving $f'(3)$ when $f(x) = x^2 + 5x$ have been shuffled. Please number the equations with the numbers 1 through 8, indicating the order in which they would appear had someone correctly calculated $f'(3)$.”
- Exercise 2: “The steps for calculating $f'(2)$ when $f(x) = 3x^2 - x$ are in the correct order. However, at each step, you are presented with two choices: a correct expression and an incorrect expression. For each step, please draw a box around the correct expression, and provide a brief explanation of what is wrong with the incorrect expression.”
- Exercise 3: “Please fill in the missing information and details on some of the steps in calculating $f'(4)$, for $f(x) = \sqrt{x}$.”
- Exercise 4: “Please use the general procedure illustrated in the above exercises to find $f'(3)$, if $f(x) = \frac{1}{x}$.”

Leading questions and general ideas. As the students explore this activity, certain questions, like the following, may arise—or you may wish to bring them up to guide the students in their learning.

-
-
-
-
-

Debrief. If possible, leave some time after the activity is completed for questions, and for discussion of the facts, procedures, and ideas that the activity was meant to reinforce.

Here are some possible takeaways from this activity:

-
-
-
-
-

Follow-up challenge.