Definite Integral Dominoes

Purpose. This activity is intended to get students thinking about, and discussing, the computation of basic definite integrals and related geometric representations.

Preparation (before class) and implementation (in class). This activity should take 15-25 minutes, depending on the class and the time reserved for follow-up discussion.

There are twelve dominos in the set. Each domino includes a definite integral (with boundaries from 0 to 1) on the top half of each domino, and a numerical or graphical solution on the bottom half. Each page should be printed on white paper or card stock and, if practical, laminated. Each domino should be cut along the solid lines, leaving the dashed lines intact.

In class, each small group should receive a shuffled set of twelve domino cards. They should then be instructed to organize the twelve cards into a chain, in which the definite integral on one domino is matched with a numerical or graphical solution found on the next domino in the chain. (Alternatively, if you wish to make this project as discovery-based as possible, you can distribute the activity—or have it waiting on students' tables as they come in—without instructions.)

It is expected that students will need to compute many of the integrals by hand. However, the integration does not require any such advanced methods as integration by parts or substitution. Students will likely compute more integrals than they need to, since only half the definite integrals have matching numerical answers. As they match integrals with related graphs, the discussion will likely turn to critical values and characteristics of the given functions.

It is likely that some students will be challenged by the computation, particularly if their algebra skills are limited, or if they do not recognize the algebraic relationships in the rational and exponential functions. There are also subtle differences between a few of the graphical solutions.

Leading questions and general ideas. As the students explore this activity, certain questions, including but not limited to the following, may arise—or you may wish to bring them up to guide the students in their learning.

- What strategies did you use to match the integrals to the solutions? Which integrals were easy to match, and which were more challenging? Why?
- Were there any errors that you made in computing numerical solutions that you were able to identify and correct? Describe some of the errors that you or your group made.
- Did you notice any relationships among any of the integrals? Explain.
- Describe one or two strategies you found helpful to complete the chain.

Debrief. If possible, leave some time after the activity is completed for questions, and for discussion of the facts, procedures, and ideas that the activity was meant to reinforce.

Here are some possible takeaways from this activity:

- Definite integrals can be represented as an area.
- Definite integrals can be evaluated using the Fundamental Theorem of Calculus (the "Evaluation theorem").
- It is often valuable to simplify the expression for a function before antidifferentiating.
- A function can often be identified by examining its values at specific points (for example, this strategy can be used to distinguish $\sqrt[3]{x}$ from $\sqrt{x}$, or to confirm the graph of e^{x+1})
- Is a u and du substitution essential to evaluate any of these definite integrals, or can each of them be antidifferentiated in some other way?