Circadian (meaning "every-24-hour") rhythms are produced in fruit flies by the interactions of messenger RNA, or mRNA, denoted M, with certain protein monomers, denoted P_1, and protein dimers, denoted P_2. (A monomer is a basic protein building block; a dimer is composed of two monomers.) These interactions proceed according to the following assumptions.

- In the absence of proteins, mRNA grows at a constant rate. However, this rate is inhibited by the presence of dimers – the larger the concentration of dimers, the more slowly mRNA grows.
- mRNA degrades (decays) at a rate proportional to the amount of mRNA present.
- Protein monomers are produced at a rate proportional to the amount of mRNA present.
- Through a process called *proteolysis*, monomers decay at a rate proportional to the amount of monomers present, and dimers decay at a rate proportional to the amount of dimers present.
- Through a process called *phosphorylation*, monomers decay – or, technically, are inactivated – upon combining with phosphates. The larger the concentration of dimers, compared to that of monomers, the more slowly monomers decay.
- Dimers are also inactivated through phosphorylation. Here, the larger the concentration of monomers, compared to that of dimers, the more slowly dimers decay.
- A dimer can split to form two monomers. This happens, at any given point in time, at a rate proportional to the concentration of dimers at that point.
- Two monomers can join to form a dimer. This happens, at any given point in time, at a rate proportional to the number of monomer-to-monomer reactions possible at that point.

Put the cards together to form a system of differential equations modeling the above interactions.

(All lower-case letters, except for t, denote positive parameters.)

$$\frac{dM}{dt} =$$

$$-\frac{eP_1}{f + P_1 + gP_2}$$

$$-nP_2$$

$$+pM$$

$$+kP_1^2$$

$$-2kP_1^2$$

$$+\frac{a}{1 + bP_2^2}$$

$$+2\ell P_2$$

$$-\frac{mP_2}{f + P_1 + gP_2}$$

$$-cM$$

$$-hP_1$$

$$\frac{dP_2}{dt} =$$

$$-\ell P_2$$

$$\frac{dP_1}{dt} =$$

$$+a$$

$$-2qP_1^2$$

Answer key

$$\frac{dM}{dt} = +\frac{a}{1+bP_2^2} - cM$$

$$\frac{dP_1}{dt} = +pM - \frac{eP_1}{f+P_1+gP_2} - hP_1 - 2kP_1^2 + 2\ell P_2$$

$$\frac{dP_2}{dt} = -\frac{mP_2}{f+P_1+gP_2} - nP_2 + kP_1^2 - \ell P_2$$