Wine is produced by adding yeast to grape juice. The yeast Y digests the sugar S in the juice, yielding alcohol A as a by-product. The alcohol is, in turn, toxic to the yeast. In this way, the yeast, alcohol, and sugar interact until equilibrium (wine!) is attained.

Assumptions:

- A pound of sugar can sustain at most b pounds of yeast. If the sugar-to-yeast ratio is much smaller than b, then yeast grows roughly exponentially, but as the sugar-to-yeast ratio gets close to b, the rate of growth of yeast gets close to zero.
- Yeast dies at a rate proportional to both the amount of alcohol present and the amount of yeast present.
- Alcohol is produced at a rate proportional to the amount of yeast present.
- Sugar is consumed at a rate proportional to the amount of yeast present.

Put the cards together to form a system of differential equations modeling the above interactions.

(All lower-case letters, except for t, denote positive parameters.)

$\dfrac{dY}{dt} =$	$\dfrac{dA}{dt} =$	$\dfrac{dS}{dt} =$	$\dfrac{dW}{dt} =$
$k\,Y\left(1 - \dfrac{Y}{b\,S}\right)$	$\dfrac{Y}{bS}$	$-f\,Y$	
$-f\,W$	$e\,Y$	$-e\,S$	$k\,Y\left(1 - b\,\dfrac{Y}{S}\right)$
$-h\,S$	$\dfrac{k\,Y}{1 + b\,S}$	$m\,W$	$q(A + Y)$

Answer key

$\frac{dY}{dt} =$	$k\,Y\left(1 - \frac{Y}{b\,S}\right)$	$-c\,A\,Y$
$\frac{dA}{dt} =$	$e\,Y$	
$\frac{dS}{dt} =$	$-f\,Y$	